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MULTI-GRID FOR STRUCTURES ANALYSIS

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Introduction

In structural analysis the amount of computational time necessary for a solution is proportional to the number of degrees of freedom times the bandwidth squared. In implicit time analysis, this must be done at each discrete point in time. If, in addition, the problem is nonlinear, then this solution must be iterated at each point in time. If the bandwidth is large, the size of the problem that can be analyzed is severely limited.

The multi-grid method is a possible algorithm which can make this solution much more computationally efficient. This method has been used for years in computational fluid mechanics. It works on the fact that relaxation is very efficient on the high frequency components of the solution (nearest neighbor interactions) and not very good on low frequency components of the solution (far interactions). The multi-grid method is then to relax the solution on a particular model until the residual stops changing. This indicates that the solution contains the higher frequency components. A coarse model is then generated for the lower frequency components to the solution. The model is then relaxed for the lower frequency components of the solution. These lower frequency components are then interpolated to the fine model.

In computational fluid mechanics the equations are usually expressed as finite differences. To generate a coarse model, the grid size is just doubled and a Green's integral theorem is used to obtain the forcing function on the coarse grid. To transfer the lower frequency solution back to the fine grid, linear interpolation is used.

In structural dynamics the equations are usually expressed as finite elements. Neighbor elements need not be connected. The process of condensing a fine model into a coarse model and interpolating the low frequency solution to the fine model is not clear.

OBJECTIVE: IMPLICIT TIME MARCH SOLUTION OF NONLINEAR STRUCTURAL DYNAMICS

- o BEAM (MULTI-SHAFT, COMBINED LATERAL, TORSIONAL, AND AXIAL ANALYSIS)
- o PLATE (BLADE VIBRATION WITH COULOMB DAMPING))
- o 3-D (SPACE STRUCTURES ANALYSIS)

Objective

The objective of this work is to use an implicit time march solution to study nonlinear structural dynamics. The work will be done in three phases. The first phase will be a beam structure. This will have application in a multi-shaft, combined lateral, torsional, and axial rotor dynamic analysis. The second phase will be a plate structure. This will have application in bladed disk vibration with Coulomb damping. The third phase will be a full 3-D structure. This will have application in space structures.

NUMERICAL INTEGRATION

LET $R(t)$ BE AN n ELEMENT VECTOR OF NODAL DISPLACEMENTS AND

$$V(t) = \dot{R} \quad , \quad A(t) = \dot{V}$$

MODIFIED TAYLOR SERIES

$$R(t) = R(0) + V(0)t + \frac{1}{2} A(0)t^2 + \frac{1}{6} \alpha \dot{A}(\xi)t^3$$

$$V(t) = V(0) + A(0)t + \frac{1}{2} \beta \dot{A}(\xi)t^2$$

$$A(t) = A(0) + \dot{A}(\xi)t$$

WHERE

$$0 \leq \xi \leq t$$

AND; α AND β ARE DETERMINED SO THAT THE METHOD IS NUMERICALLY STABLE AS $t \rightarrow \infty$

Numerical Integration

The numerical integration method is based on a Nordsieck-like method. The displacement, velocity, and acceleration are defined at an initial time. A modified Taylor series is used to calculate the displacement, velocity, and acceleration at the advanced time. The Lagrange's remainder term, the time derivative of the acceleration, is calculated from the equations of motion at the advanced time. Alpha and beta are constants determined so that the method is stable as time approaches infinity.

This method of integration for a first order differential equation is Gear's method (Gear, 1971). Zeleznik (1979) showed that this method could be used on higher order equations. Kascak (1980) showed that for a third order integrator used on a linear second order differential equation the method is unconditionally stable.

NUMERICAL STABILITY

$$MA + CV + KR = F$$

$$\left(tM + \frac{1}{2}\beta t^2 C + \frac{1}{6}\alpha t^3 K\right)\dot{A}(\xi) = F - \left(M + tC + \frac{1}{2}t^2 K\right)A(0) - (C + tK)V(0) - KR(0)$$

AS $t \to \infty$ $\qquad \dot{A}(\xi) \approx -\left(\frac{3}{\alpha t}\right)A(0)$

$$R = R(0) + V(0)t$$

$$V = V(0) + \left(1 - \frac{3}{2}\left(\frac{\beta}{\alpha}\right)\right)A(0)t$$

$$A = \left(1 - \left(\frac{3}{\alpha}\right)\right)A(0)$$

LET $\alpha = 3$ AND $\beta = 2$ $\therefore$ $R = R(0) + V(0)t$

$$V = V(0)$$

$$A = 0$$

Numerical Stability

The numerical stability of this method can be examined by substituting the displacement, velocity, and acceleration into the linear equations of motion and solving for the time derivative of the acceleration. As time approaches infinity the dominate term on both the right and left side of the equation has the stiffness matrix as a pre-multiplier. The time derivative of the acceleration is proportional to the initial acceleration divided by the time. If this is substituted into the modified Taylor series, and alpha is set to 3 and beta is 2: the acceleration is zero and the velocity is constant. The eigenvalues become zero and one.

ITERATIVE SOLUTION

GIVEN: $R(0)$, $V(0)$, $A(0)$, AND $\dot{A}(\xi) \sim \dot{A}^{(0)}$

THEN: $R^{(0)} = R(0) + V(0)t + \frac{1}{2} A(0)t^2 + \frac{1}{2} \alpha \dot{A}^{(0)} t^3$

$V^{(0)} = V(0) + A(0)t + \frac{1}{2} \beta \dot{A}^{(0)} t^2$

$A^{(0)} = A(0) + \dot{A}^{(0)} t$

LET: $\dot{A}(\xi) = \dot{A}^{(0)} + \Delta \dot{A}$

THEN: $R(t) = R^{(0)} + \frac{1}{6} \alpha \Delta \dot{A} t^3$

$V(t) = V^{(0)} + \frac{1}{2} \beta \Delta \dot{A} t^2$

$A(t) = A^{(0)} + \Delta \dot{A} t$

Iterative Solution

If the initial displacement, velocity, acceleration, and an initial estimate of the time derivative of the acceleration are given, then an estimate of the advanced displacement, velocity, and acceleration is given by the modified Taylor series. The correction to the time derivative of the acceleration can be found from the equations of motion.

NONLINEAR EQUATION OF MOTION

$$0 = F(R, V, A, t)$$

WHERE F IS AN n ELEMENT VECTOR SUM OF THE STATIC AND DYNAMIC FORCES

THEN: $$0 = F\left(R^{(0)} + \frac{1}{6}\alpha\Delta\dot{A}t^3,\ V^{(0)} + \frac{1}{2}\beta\Delta\dot{A}t^2,\ A^{(0)} + \Delta\dot{A}t,\ t\right)$$

OR: $$0 = F(\Delta\dot{A})$$

Nonlinear Equations of Motion

The nonlinear equations of motion are the sum of both the static and dynamic forces for each element. As such, the equations are functions of the displacement, velocity, acceleration, and time. If the modified Taylor series is substituted into the equations of motion using the iterative form, then the equations of motion become a function of the correction to the time derivative of the acceleration.

LINEARIZED EQUATION OF MOTION

$$0 = F^{(0)} - B\Delta\dot{A}$$

WHERE

$$F^{(0)} = F(R^{(0)}, V^{(0)}, A^{(0)}, t)$$

$$B = \frac{1}{6}\alpha t^3 K + \frac{1}{2}\beta t^2 C + tM$$

$$K = -\frac{\partial F}{\partial R}, \quad C = -\frac{\partial F}{\partial V}, \quad M = -\frac{\partial F}{\partial A}$$

$$\therefore \; B\Delta\dot{A} = F^{(0)}$$

Linearized Equations of Motion

To solve for the correction, the equations of motion are linearized about the estimated values. The instantaneous stiffness, damping, and mass are defined by the various partial derivatives wth respect to displacement, velocity, and acceleration. If the linearization is done numerically, the stiffness, damping, and mass don't have to be calculated. The numerical differentiation of the correction to the time derivative of the acceleration is all that is needed.

This solution procedure is equivalent to the Newton-Raphson technique. The numerical differentiation and the solution of the linearized equations of motion are computationally time consuming, although straight-forward. The multi-grid technique could potentially be orders of magnitude faster. The linearized equations of motion will be the basis for generating a coarse model from a fine model.

STATIC CONDENSATION

$$\left[\begin{array}{c|c} B_{11} & B_{12} \\ \hline B_{21} & B_{22} \end{array}\right]\left[\begin{array}{c} \Delta\dot{A}_1 \\ \hline \Delta\dot{A}_2 \end{array}\right] = \left[\begin{array}{c} F_1^{(0)} \\ \hline F_2^{(0)} \end{array}\right]$$

$$\left(B_{11} - B_{12}B_{22}^{-1}B_{21}\right)\Delta\dot{A}_1 = F_1^{(0)} - B_{12}B_{22}^{-1}F_2^{(0)}$$

$$\Delta\dot{A}_2 = B_{22}^{-1}\left(F_2^{(0)} - B_{21}\Delta\dot{A}_1\right)$$

LET $B^{(1)} = B_{11} - B_{12}B_{22}^{-1}B_{21}, \; F^{(1)} = F_1^{(0)} - B_{12}B_{22}^{-1}F_2^{(0)}$

$$\therefore B^{(1)}\Delta\dot{A}_1 = F^{(1)}$$

IF $F_2^{(0)} = 0 \;\Rightarrow\; \Delta\dot{A}_2 = -B_{22}^{-1}B_{21}\Delta\dot{A}_1$ (INTERPOLATOR)

Structural Condensation

If the linearized equation set is partitioned into nodes belonging to a coarse model (the top partition) and the nodes eliminated from the fine model (the bottom partition), then structural condensation can be used to solve for the coarse model. In addition, the structural condensation process can be used to interpolate the solution from the coarse model to the fine model. If the higher frequency part of the solution is found on the fine model and the lower frequency part of the solution is found on the coarse model, then the resultant forces must be zero. Thus the solution for the nodes eliminated from the fine model can be found.

FINE-TO-COARSE MODEL TRANSFORMATION

$$\Phi = \left[\begin{array}{c} I \\ \hline -B_{22}^{-1}B_{21} \end{array}\right] \Rightarrow \Delta\dot{A} = \Phi\Delta\dot{A}_1$$

COARSE-TO-FINE MODEL TRANSFORMATION

$$\theta = \left[\begin{array}{c:c} I & -B_{12}B_{22}^{-1} \end{array}\right] \Rightarrow F^{(1)} = \theta F^{(0)}$$

$$\therefore\ B\dot{A} = F^{(0)} \Rightarrow \theta B\Phi\Delta\dot{A}_1 = \theta F^{(0)}$$

OR

$$B^{(1)}\Delta\dot{A}_1 = F^{(1)}$$

Fine-to-Coarse and Coarse-to-Fine Model Transformations

The fine-to-coarse model transformation is a rectangular matrix which averages the force from the fine model to the coarse model. The upper partition is an identity matrix and the lower partition is defined in the structural condensation process. The coarse-to-fine transformation interpolates the correction of the time derivative of the acceleration from the coarse to fine model. In the symmetric case, the fine-to-coarse transformation is the transpose of the coarse-to-fine transformation.

NONLINEAR CONDENSATION

$$0 = F(\Delta\dot{A}) \Rightarrow 0 = \theta F(\Phi\Delta\dot{A}_1)$$

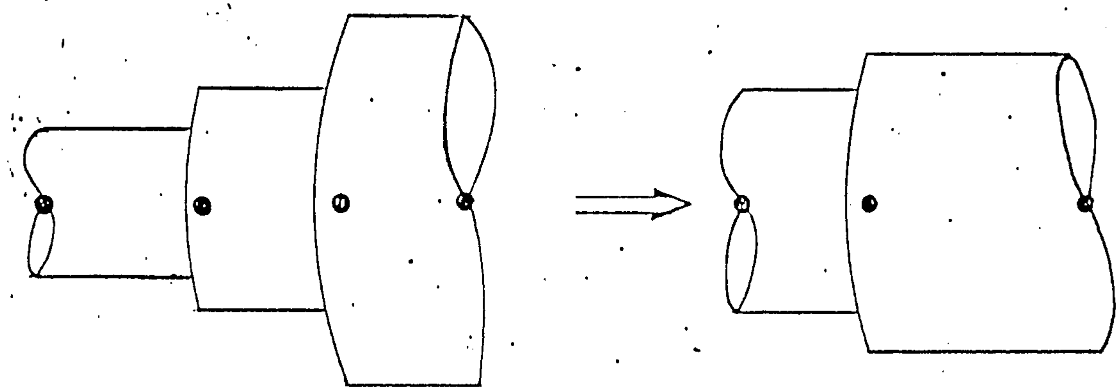
RELAXATION

$$0 = \theta F^{(0)} - D\Delta\dot{A}_1 \Rightarrow \Delta\dot{A}_1 = D^{-1}F^{(1)}$$

Nonlinear Condensation

The nonlinear condensation process transforms the independent variables from the coarse model to the fine model and the dependent variables from the fine to coarse model. Thus the resultant forces are relaxed on the coarse model. This would only require the inversion of a diagonal matrix. The corrections on the coarse model are then interpolated to the fine model. The linearization of the equations of motion is not needed in the solution process, but only needed to define the transformations.

LOCAL STRUCTURAL CONDENSATION



B IS BLOCK TRIDIAGONAL -- INCLUDES NEAREST NEIGHBOR INTERACTION, NEGLECT FAR INTERACTION

Local Structural Condensation

The linearization of the equations of motion and the structural condensation process require a considerable amount of computational time. Multi-grid via relaxation is most efficient on nearest neighbor interactions. Thus only a partial linearization of the equations of motion is necessary. The equations of motion only have to be linearized with respect to the node under consideration and its nearest neighbors. Applying condensation to this local interaction model results in local structural condensation. In the case of a beam, this linearization results in a block tridiagonal matrix and the structural condensation results in a coarse model in which every other node is removed from the fine model.

$$
\begin{pmatrix}
V_1 W_1 & & & & & & & & \\
U_2 V_2 W_2 & & & & & & & \bigcirc & \\
 & U_3 V_3 W_3 & & & & & & & \\
 & & U_4 V_4 W_4 & & & & & & \\
 & & & U_5 V_5 W_5 & & & & & \\
 & & & & U_6 V_6 W_6 & & & & \\
\bigcirc & & & & & U_7 V_7 W_7 & & & \\
 & & & & & & U_8 V_8 W_8 & & \\
 & & & & & & & U_9 V_9 &
\end{pmatrix}
\begin{pmatrix}
\Delta\dot{A}_1 \\ \Delta\dot{A}_2 \\ \Delta\dot{A}_3 \\ \Delta\dot{A}_4 \\ \Delta\dot{A}_5 \\ \Delta\dot{A}_6 \\ \Delta\dot{A}_7 \\ \Delta\dot{A}_8 \\ \Delta\dot{A}_9
\end{pmatrix}
=
\begin{pmatrix}
F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \\ F_9
\end{pmatrix}
\qquad B\Delta\dot{A} = F^{(0)}
$$

$$
\left(\begin{array}{cccc|ccccc}
V_2 & \bigcirc & & & U_2 W_2 & & & \bigcirc & \\
 & V_4 & & & & U_4 W_4 & & & \\
\bigcirc & & V_6 & & \bigcirc & & U_6 W_6 & & \\
 & & & V_8 & & & & U_8 W_8 & \\
\hline
W_1 & \bigcirc & & & V_1 & & & \bigcirc & \\
U_3 W_3 & & & & & V_3 & & & \\
 & U_5 W_5 & & & & & V_5 & & \\
\bigcirc & & U_7 W_7 & & \bigcirc & & & V_7 & \\
 & & & U_9 & & & & & V_9
\end{array}\right)
\left(\begin{array}{c}
\Delta\dot{A}_2 \\ \Delta\dot{A}_4 \\ \Delta\dot{A}_6 \\ \Delta\dot{A}_8 \\ \hline \Delta\dot{A}_1 \\ \Delta\dot{A}_3 \\ \Delta\dot{A}_5 \\ \Delta\dot{A}_7 \\ \Delta\dot{A}_9
\end{array}\right)
=
\left(\begin{array}{c}
F_2 \\ F_4 \\ F_6 \\ F_8 \\ \hline F_1 \\ F_3 \\ F_5 \\ F_7 \\ F_9
\end{array}\right)
\qquad
\left(\begin{array}{cc}
B_{11} & B_{12} \\ \hline
B_{21} & B_{22}
\end{array}\right)
\left(\begin{array}{c}
\Delta\dot{A}_1 \\ \hline \Delta\dot{A}_2
\end{array}\right)
=
\left(\begin{array}{c}
F_1^{(0)} \\ \hline F_2^{(0)}
\end{array}\right)
$$

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Beam Example

If the tridiagonal equation set is re-ordered into even-numbered in both the fine and coarse model (top) and odd-numbered just in the fine model (bottom), then the structural condensation has a simple form. In the re-ordered equation set, the block matrices on the diagonal are diagonal. The inversion of these block matrices is trivial.

$$-B_{22}^{-1}B_{21} = \begin{pmatrix} T_1 & & \bigcirc & \\ S_2 & T_2 & & \\ & S_3 & T_3 & \\ & & S_4 & T_4 \\ \bigcirc & & & S_5 \end{pmatrix} \qquad \begin{array}{l} T_L = -V_{2L-1}^{-1}W_{2L-1} \\ \\ S_L = -V_{2L-1}^{-1}U_{2L-1} \end{array}$$

$$-B_{12}B_{22}^{-1} = \begin{pmatrix} X_1 & Y_1 & & \bigcirc & \\ & X_2 & Y_2 & & \\ & & X_3 & Y_3 & \\ \bigcirc & & & X_4 & Y_4 \end{pmatrix} \qquad \begin{array}{l} X_L = -U_{2L}V_{2L-1}^{-1} \\ \\ Y_L = -W_{2L}V_{2L+1}^{-1} \end{array}$$

$$B_{11} - B_{12}B_{22}^{-1}B_{21} = \begin{pmatrix} V_1^{(1)} & W_1^{(1)} & & \bigcirc \\ U_2^{(1)} & V_2^{(1)} & W_2^{(1)} & \\ & U_3^{(1)} & V_3^{(1)} & W_3^{(1)} \\ \bigcirc & & U_4^{(1)} & V_4^{(1)} \end{pmatrix} \qquad \begin{array}{l} V_L^{(1)} = V_{2L} + X_L W_{2L-1} + Y_L U_{2L+1} \\ \\ U_L^{(1)} = +X_L U_{2L-1} \\ \\ W_L^{(1)} = +Y_L W_{2L+1} \end{array}$$

$$\Delta\dot{A}_{2L} = \Delta\dot{A}_L^{(1)} \quad , \quad \Delta\dot{A}_{2L-1} = S_L \Delta\dot{A}_{L-1}^{(1)} + T_L \Delta\dot{A}_L^{(1)}$$

$$F_L^{(1)} = F_{2L} + X_L F_{2L-1} + Y_L F_{2L+1}$$

Solution of Beam Example

The solution for the non-identity partition of both transformations is tridiagonal. The non-identity partition of the fine-to-coarse transformation is also lower triangular. The non-identity partition of the coarse-to-fine transformation is also upper triangular.

ACCALERATION PARAMETER

$\epsilon \, (\, |\lambda| \,)$ - based on local coefficients

$$\lambda = \frac{(\Delta \dot{\vec{A}})^T \, D \, (\Delta \dot{\vec{A}})}{(\Delta \dot{\vec{A}})^T \, (\Delta \dot{\vec{A}})}$$

(Rayleigh Quotient)

Acceleration Parameter

Normally the relaxation technique can be improved by using a weighted average of the previous and calculated values of the corrections to the solution (over relaxation). The rate of convergence of the high frequency components can be improved at the expense of the low frequency components. To do this an estimate of highest frequency eigenvalue is needed. The Rayleigh quotient is a good method to estimate the highest eigenvalue (at least in the symmetric case). In addition, the highest eigenvalue should be a strong function of the nearest neighbors; therefore, local linearization could be used in the Rayleigh quotient.

MULTI GRID METHOD

- RELAX ON FINE GRID TO GET HIGH FREQUENCY COMPONENT
- CALCULATE RESIDUAL ON FINE GRID
- CHECK RESIDUAL FOR SOLUTION
- CHECK CHANGE IN RESIDUAL FOR CHANGE IN GRID
- STATIC CONDENSE TO COARSE GRID
- RELAX ON COARSE GRID TO GET LOW FREQUENCY COMPONENT
- INTERPOLATE LOW FREQUENCY TO FINE GRID

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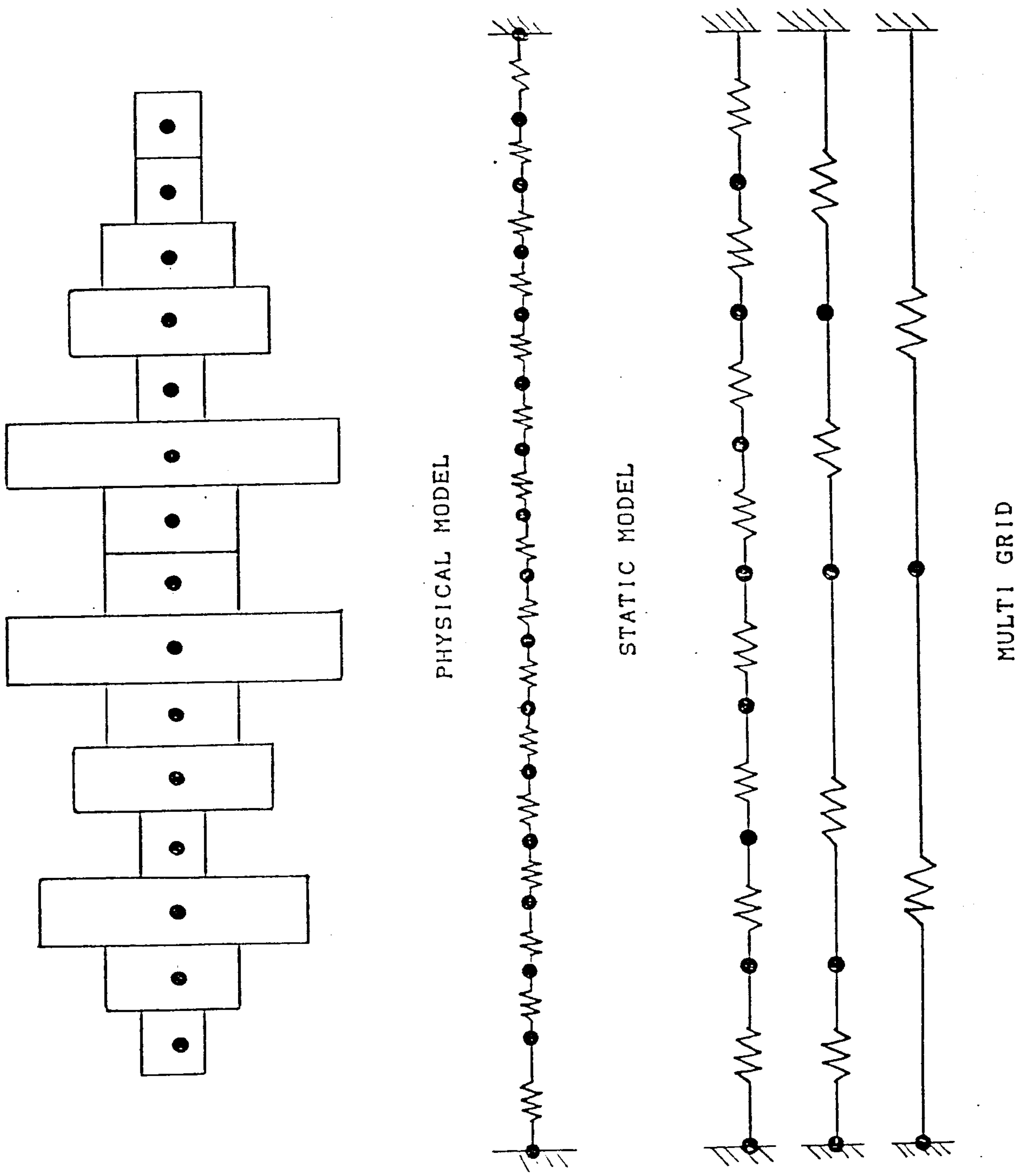

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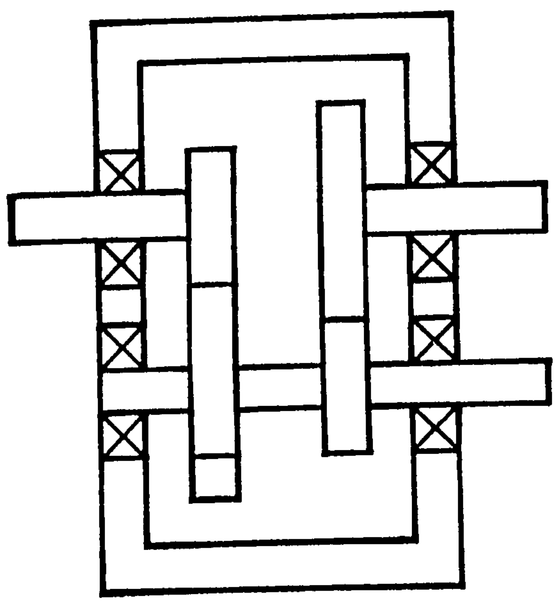
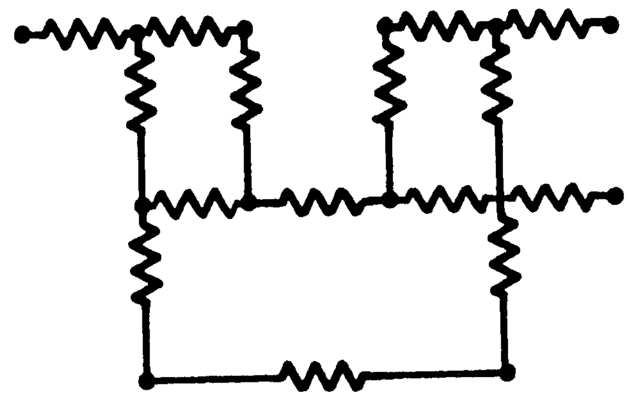
Multi-Grid Method

In summary, the multi-grid method for structural dynamics is first to relax the equations of motion on the fine grid to obtain the high frequency components of the solution. Then calculate the norm of the residual on the fine model. Next check to see if the norm is small enough for a solution. If not, check to see if the norm has changed significantly from the previous iteration. If the norm has changed, then relax the solution until the norm stops changing. This indicates that the high frequency components on this model have been found.

To find the lower frequency components of the solution, local structural condensation is used to generate a coarse model. On the coarse model, relaxation is used to generate the lower frequency components of the solution. These lower frequency components are interpolated to the fine grid where the norm of the residual is calculated. Based on this norm, either a solution is found, more relaxation is needed, or a coarser model is needed. The process is repeated until a solution is found.



PHYSICAL MODEL



NONLINEAR FINITE ELEMENT MODEL

Multi-Grid Analysis Applied to Transmission Dynamics

Transmission dynamics is a case of nonlinear structural dynamics. Physically a transmission is composed of gears, shafts, bearings, seals, and a case. The case and the shafts can be modeled by finite element methods. The bearings and seals are modeled by special programs developed in tribology and other areas. Gear interactions are developed for some kind of gears, but not for others. Thus, a transmission can be modeled by a number of linear and nonlinear finite elements. As a first approximation, a transmission can be modeled as a beam structure. The transmission can be analyzed as a multi-shaft, combined lateral, torsional, and axial rotor dynamic system.

SPECIAL ASPECTS

- Gyroscopic and gear forces cause nonlinear lateral and torsional coupling.

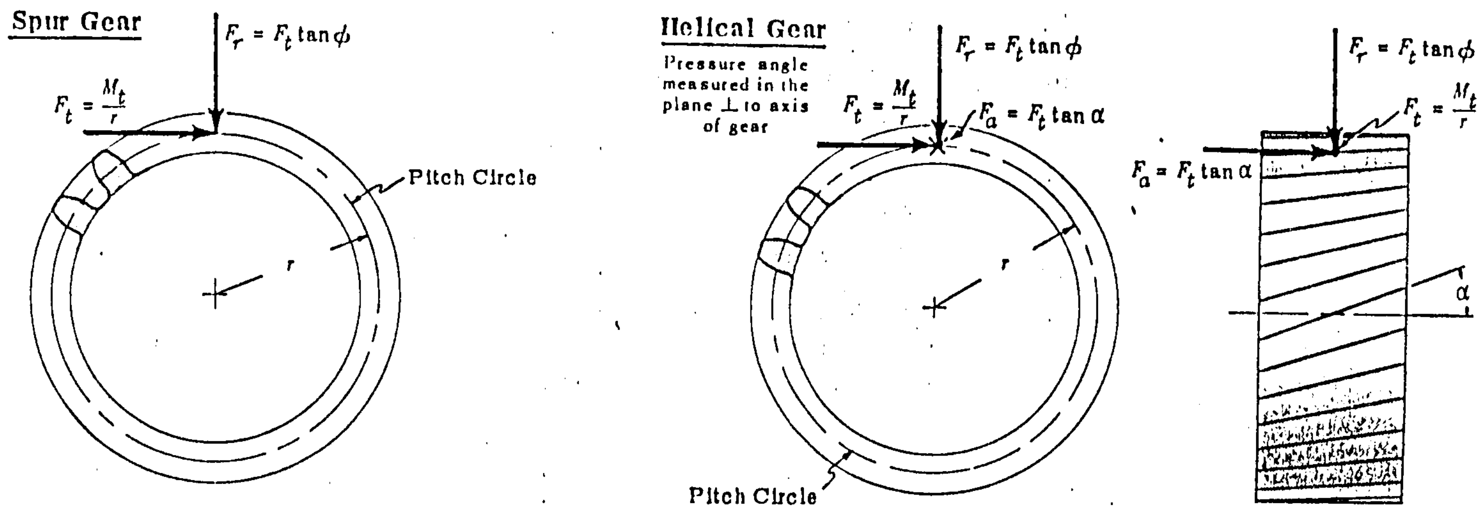
- Gear tooth pass frequencies are high frequency forcing functions. This implies a need for a fine structural model.

- Gear - gear interactions cause a wide band width.

Special Aspects

Special aspects complicate the dynamic analysis of transmissions. Gyroscopic and gear forces cause nonlinear lateral and torsional coupling. Gear tooth passing frequencies are high frequency forcing functions. This implies a need for a fine structural model. Gear-gear interactions cause the system to have a wide bandwidth.

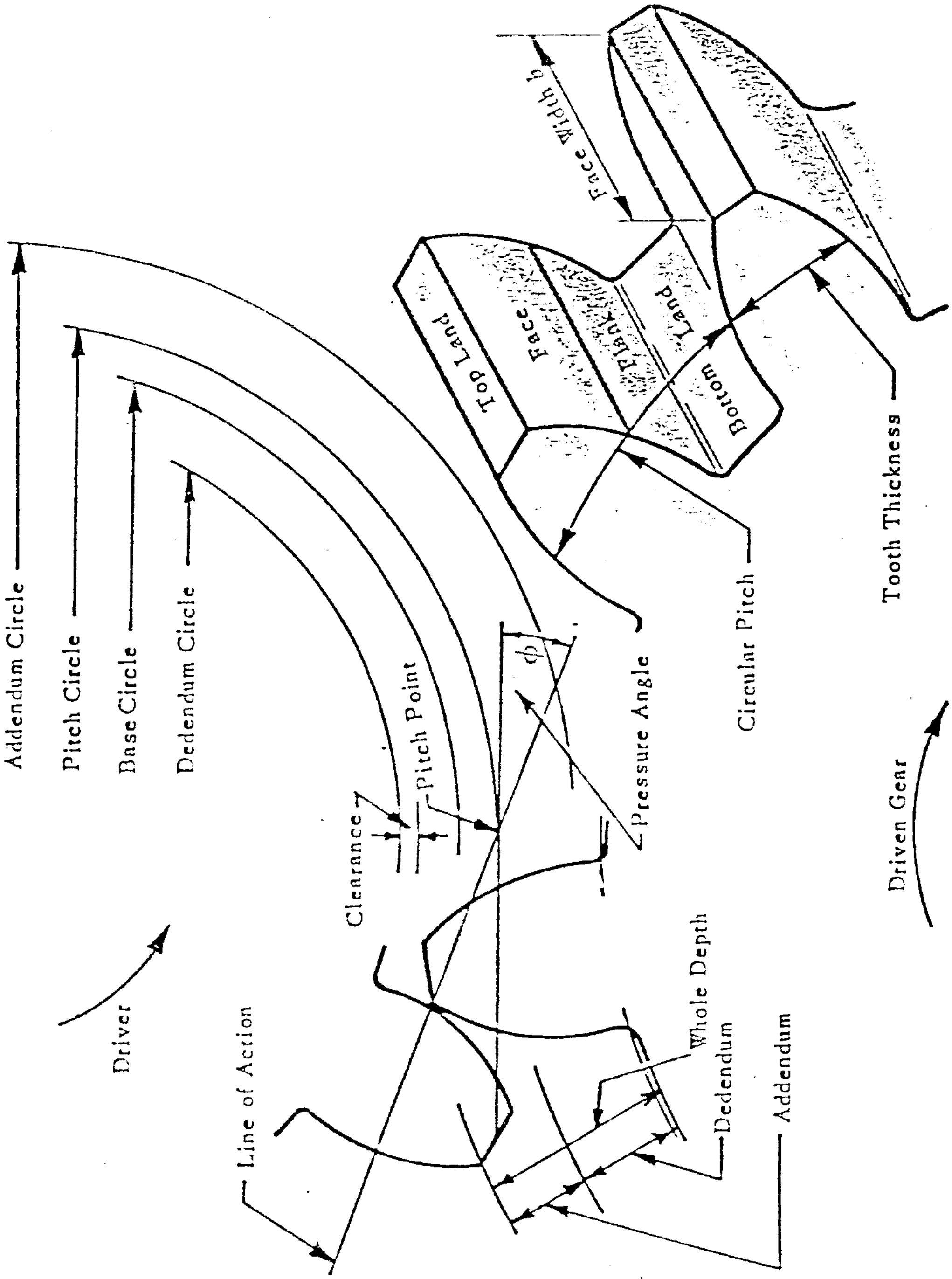
GEAR TOOTH INTERACTION



Gear Tooth Interaction

Consider gear tooth interaction. For any gear set the line of force does not pass through the gear centers. In the case of spur gears any perturbation of the radial force will result in a perturbation of the tangential force and vice versa. In the case of helical or spiral gears any perturbation of the radial force will result in perturbations of both the axial and tangential forces. This results in a nonlinear coupling between the axial, tangential, and radial directions.

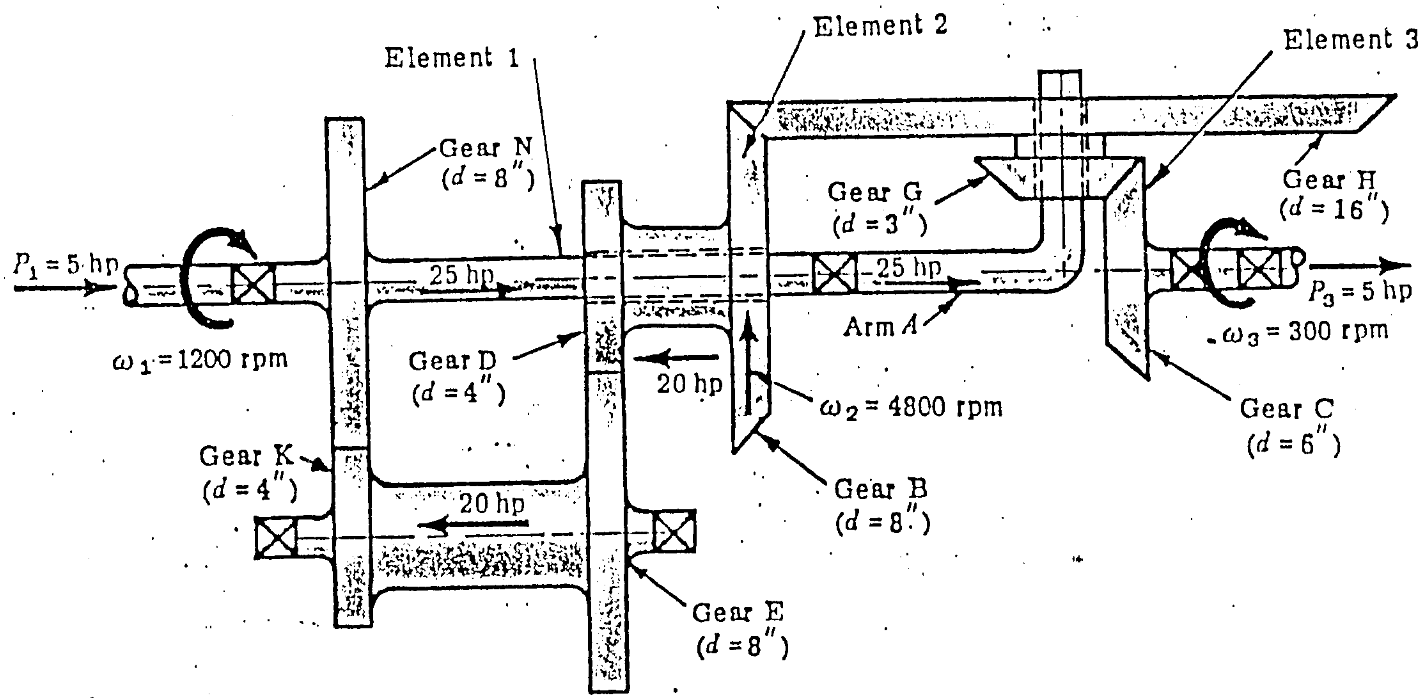
GEAR TOOTH MODEL



Gear Tooth Model

Consider the gear tooth interactions of a spur gear set. The contact point varies as the angle of the gear set varies. Machining errors cause the contact point to move. High torque can cause the teeth to bend. The number of teeth in contact varies as the torque varies. Negative torque can result in backlash. The force must be transmitted through the contact point. All these effects cause nonlinear time varying interactions between the spur gears set. For the other kind of gears the interaction is more complicated. Thus, gear tooth interactions cause high frequency forcing functions on the structure.

TYPICAL TRANSMISSION



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Typical Transmission

In a typical transmission there are many gear sets. Each of these gear sets causes one location on the structure to interact with another point on the structure. Thus, far interactions are important and the structural model has a wide bandwidth.

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Transmission Dynamic Analysis

Complete transmission dynamic analyses are rare in the open literature. David and Mitchell (1986) have used a modal balance technique. The problem with modal techniques is that the nonlinearities cause the set of modes not to be closed. This results in side bands around the tooth passing frequency. Whenever a solution is found, it is not known whether all of the important modes in the solution have been included. Also, superfluous modes tend to overwhelm the solution technique. The time march multi-grid method should eliminate these problems.

APPLICATION OF MULTIGRIDDING TECHNIQUES TO STRUCTURAL ANALYSIS USING A PARALLEL TRANSPUTER ARRAY

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Application of Multi-Grid Techniques To Structural Analysis Using a Parallel Transputer Array

A large number of multi-grid computational steps can be done in parallel. A new parallel computing system based on the transputer chip has recently become available. The transputer chip is a self-contained high performance computer. Separate processors within the transputer chip perform normal computations, manage memory, perform floating point arithmetic functions, and manage communications with other transputers concurrently. Many transputers can be linked together to form a parallel processing computer network by simply connecting serial communication links between transputers. A team has been assembled to apply the multi-grid technique to this transputer array.

OBJECTIVE

TIME MARCH SOLUTION OF NONLINEAR STRUCTURAL DYNAMICS

- O BEAM (MULTI-SHAFT - COMBINED LATERAL, TORSIONAL AND AXIAL ANALYSIS)
- O PLATE (BLADE VIBRATION WITH COULOMB DAMPING)
- O 3-D (SPACE STRUCTURES ANALYSIS)

RELATED WORK

- O MULTIGRID ANALYSIS USED IN FLUID DYNAMICS BUT NOT IN STRUCTURAL DYNAMICS
- O TRANSPUTER USED IN STRUCTURAL STATICS, (SPARTA)
- O TRANSPUTER USED IN GRAPHICAL DISPLAY, G. K. ELLIS (ICOMP)

Objective

The objective of this research is to take the serial codes developed for multi-grid technique applied to the beam, plate, and 3-D brick elements and apply them to the parallel transputer array.

Related Work

The multi-grid analysis has been used in fluid dynamics for years, but not in structural dynamics. The transputer is currently being applied to static structural problems using a direct or wave front technique. In addition, the transputer is being used to process graphical displays. This graphical display work is being done on a transputer test bed system. The test bed system is designed to be electronically reconfigured into a variety of different equivalent architectures so that the interplay between algorithms and architectures can be fully explored.

APPROACH

USE "OCCAM" PARALLEL PROGRAMMING LANGUAGE AND ELECTRONICALLY RE-CONFIGURE ARCHITECTURE TO ASSESS VARIOUS APPROACHES TO STRUCTURAL MULTIGRID ANALYSIS

- NUMERICAL STABLE TIME INTEGRATION
- LOCAL STRUCTURAL CONDENSATION TO OBTAIN INTERPOLATION FROM COARSE TO FINE MODEL AND AVERAGE FINE TO COARSE MODEL
- RELAX EACH NODE WITH ONE CPU FOR EACH DEGREE OF FREEDOM

Approach

The approach is to use an "OCCAM" parallel programming language (specifically designed for the transputer) and the test bed (with electronically reconfigurable architectures) to assess their application to structural dynamic multi-grid analysis. Areas which can profit from parallel computations are the time step advancement, the coarse-to-fine and fine-to-coarse transformations, and the relaxation process.

POTENTIAL IMPACT

- DEMONSTRATION OF A NEW APPROACH TO STRUCTURAL DYNAMIC SIMULATIONS WITH A RECONFIGURABLE PARALLEL ARCHITECTURE WHICH CAN DRAMATICALLY REDUCE THE COST AND COMPUTING TIME

Potential Impact

The multi-grid method, although used for years in fluid dynamics, now offers a new approach to nonlinear structural dynamics. The computing time does not depend on the cube of the number of degrees of freedom. Thus, dramatic reductions in computing time are possible. In addition, the relaxation process is applicable to parallel computation. Thus, the method is very attractive for future computers.

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